“Rational” Expectations, the Optimal Monetary Instrument and the Optimal Money Supply Rule

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Analyze the effects of alternative ways of conducting Monetary Policy in ad-hoc environment.

Monetary Authority:

1 – Peg interest rate period by period, Supply of money satisfies demand

2 – Set money supply period by period, accepting interest rates that equilibrate it.

2 Versions of Model

- Autoregressive version- Public‘s expectations assumed formed via fixed autoregressive schemes on variables being forecast.
- Rational Expectations version- Public’s expectations assumed equal to objective expectations that depend on rules governing monetary and fiscal policies.

Policy Implications

Rational Expectations

- A) Probability distribution of output independent of deterministic money supply rule
- B) If Loss function includes quadratic terms in the price level then optimal deterministic money supply rule is one that equates expected value of next period price level to target.
- C) Unique equilibrium price level does not exist if monetary authority pegs interest rate period-by-period, regardless how value arises.

Autoregressive version

- All the usual exploitable tradeoffs between output and inflation
- Minimization of Loss function well defined non-trivial leads to unique optimal deterministic feedback rule either for money supply or interest rate.
- Unique period-by-period equilibrium if interest rate pegged.
- Whether interest rate feedback rule or money supply feedback rule is optimal depends on most of parameters of model, including covariance matrix of disturbances

The Ad Hoc Model

Aggregate Supply:

$$Y_t = \alpha_1 k_{t-1} + \alpha_2 (p_t - {}_t p^*_{t-1}) + u_{t-1,} \quad \alpha_i > 0,\ i = 1,2 \qquad (1)$$

Aggregate Demand Schedule or IS Curve:

$$Y_t = b_1 k_{t-1} + b_2 [\, r_t - ({}_{t+1} P^*_{t-1} - {}_t P^*_{t-1})] + b_3 Z_t + u_{2t,}$$

$$b_1 > 0,\ b_2 < 0 \qquad (2)$$

– Portfolio Balance or LM schedule:

$$m_t = p_t + c_1 y_t + c_2 r_t + u_{3t} \qquad c_1>0,\ c_2<0 \quad (3)$$

– Determination of productive Capacity:

$$K_t = d_t k_{t-1} + d_2[r_t - ({}_{t+1}p^*_{t-1} - {}_t p^*_{t-1})] + d_3 Z_t + u_{4t}$$

$$d_2<0 \qquad (4)$$

Evolution of the Exogenous Variables

$$Z_t = \sum_{j=1}^{q} \rho_j Z_{t-j} + \xi_t$$

$$u_{it} = \sum_{j=1}^{q} \rho_{ij} u_{i,t-j} + \xi_{i,t} \qquad (5)$$

-Where Y_t, p_t, m_t, are the natural logarithms of output, price level and money supply, r_t is nominal rate if interest itself, Z_t is vector of exogenous variables, p* is the public's psychological expectation of the log of the price level, at end of t-j. The variable k_{t-1} is a measure of productive capacity.

2. The Stabilization Problem

The ad hoc Loss function:

$$L = E_0 \sum_{t=1}^{\infty} \delta^{t-1}[(y_t, p_t)K(y_t, p_t)' + (y_t, p_y)(K_1, K_2)' + K_1^2/4 + K_2^2/4]$$

- Function is separately quadratic in y and p
- Implies L=0, its lower bound at particular constant values of y, p
- Target values $-K_1/2K_{11}$ for y, and $-K_2/2K_{22}$ for p

Strategies

- Peg r_t via deterministic feedback rule
  - $r_t = G\theta^*_{t-1}$ (6)

Government chooses values in G to min L.

- Best money supply feedback rule of the form:
  - $m_t = H\theta^*_{t-1}$ *(7)*

$$_{t+1}P^*_t = \sum_{j=1}^{q} v_1 p_{t-I} \quad (8)$$

$$_{t+2}P^*_t = \sum_{j=1}^{q} v_2 p_{t-i} \quad (9)$$

3. The Autoregressive Expectations Version

Optimal Money Supply rule

- Assume the psychological expectations are governed by distributed-lag or "adaptive" schemes
- System formed of equations (1)-(5) and (9) can be reduced to difference equation

$$Y_{1t} = \sum_{i=1}^{q^*} A_i Y_{1t-i} + \sum_{i=0}^{q^*} B_i m_{t-i} + \phi_{1t} \qquad (10)$$

- Where to find the best money supply feedback monetary authority chooses the parameters H of the rule (7) to minimize the loss (L) subject to (10).

Optimal Interest Rate Rule

- System of equations formed by equations (1)-(5) , (8) and (9)

$$Y_{2t} = \sum_{i=1}^{q^*} C_i Y_{2t-i} + \sum_{i=0}^{q^*} D_i r_{t-i} + \phi_{2t} \qquad (11)$$

- Choose the G's of (6) to minimize L subject to(11)
- The optimal interest rate rule is the one where (1)-(5), (8) and (9) give rise to non trivial dynamic problems
- Money Supply as monetary instrument solve for (1)-(3) for y, r and p get a reduced form for p_t

$$P_t = J_o({}_tP^*_{t-1}) + J_1({}_{t+1}P^*_{t-1}) + J_2 m_t + X_t \qquad (12)$$

- Choice for deterministic part of m has effects in future periods

- With r as monetary instrument equation (2) is reduced form for y_t while for p_t is

- $\alpha_2 P_t = (\alpha_2+b_2)_t p^*_{t-1} - b_2(_{t+1}p^*_{t-1}) + b_2 r_t + (b_1-\alpha_1)k_{t-1}$

  $+b_3 Z_t - u_{1t} + u_{2t}$ (13)

- Both give rise to a non trivial dynamic problem
- Which policy is superior depends on the one that gives smaller loss, which depends on covariance structure of disturbances among other things.
- Coincides with Poole.

4. Rational Expectations under Money Supply Rule

- Impose public's expectations be rational, requiring that

$$_{t+i}p^*_{t-j} = E_{t-j}p_{t+j} \qquad (14)$$

- Available information assumed consists of data on current and past values of all endogenous variables observed as of the end t-j, ϕ^*_{t-j}
- Solve system for y, r and p given m, get pseudo reduced from for p

$$p_t = J_0 E_{t-1}p_t + J_1 E_{t-1}p_{t+1} + J_2 m_t + X_t \qquad (15)$$

- Where p_t-$E_{t-1}p_t$ is an exogenous process, unaffected by rule chosen by money supply
- Using (14) and (15) can write (1) as:

$$Y_t = \alpha_1 k_{t-1} + \alpha_2 (X_t - E_{t-1} X_t) + u_{1t} \qquad (17)$$

- Real interest rate as function of k_{t-1} and exogenous processes
- Y is an exogenous process; it has a distribution independent of deterministic rule of the money supply ((a) has been proved).

- Next to prove (b). “If Loss function includes quadratic terms in the price level then optimal deterministic money supply rule is one that equates expected value of next period price level to target.” Write *t th term of loss function*

$$L_t = E_0[\, K_{0t} + K_2 E_{t-1} p_t + K_{22} (E_{t-1} p_t)^2 \,]$$

and exogenous process

$$K_{0t} = E_{t-1}[K_{22}(p_t — E_{t-1} p_t)^2 + K_1 y_t + k_{11} y_t^2]$$

- Rule that minimizes L_t also minimizes L

$$(1—J_0)E_{t-1}p_{t+1} = \sum_{j=0}^{\infty} [J_1/(1—J_0)]^j \times E_{t-1}(X_{t+j+1} + J_2 m_{t+j+1}) \qquad (22)$$

$$(1—J_0)E_{t-1}p_t - J_1 E_{t-1}p_{t+1} = E_{t-1}X_t + J_2 m_t \qquad (23)$$

- Value of $E_{t-1}P_t$ that minimizes L_t for all t is

$$E_{t-1}P_t = -K_2/2K_{22} \qquad (24)$$

$$E_{t-1}P_{t+1} = -K_2/2K_{22} \qquad (25)$$

- Optimal rule for money supply is obtained by substituting (24) and (25) into (23) resulting in feedback in the form of equation (7).

- Definite rule emerges because when specifying L, chosen a target value for price level
- If instead L were made dependent only on variance of price level then one deterministic rule as good as any other
- Y unaffected by rule, one target, one instrument problem

Why distribution of real output is independent of money supply rule

- To induce fluctuations in real output, Gov't must induce unexpected movements in price level by virtue of AS curve.
- However by price level being rational, unexpected part of price movements is independent of systematic part of money supply as long as Authority and public share the same information
- No rule Gov't can follow that permits it to affect the unexpected part of price level
- However the Gov't can add an unpredictable random component to the systematic part of money supply

$$m_t = H\theta^*_{t-1} + \psi_t \qquad (7')$$

5. The Rational Expectations Version under an interest rate rule

- Imposing rationality, taking (14) in (13) and taking E_{t-1} of the result, then solving by recursion we get:

$$E_{t-1}P_t = -\sum_{j=0}^{n} E_{t-1}\{r_{t+j} + [(b_1 - \alpha_1)/b_2]k_{t+j-1} + (b_3/b_2) \times (Z_{t+j} - u_{1t+j} + u_{2t+j})\} + E_{t-1}p_{t+n+1} \qquad (27)$$

- When interest rate is pegged model cannot determine a path of expected prices $E_{t-1}P_{t+j}$
- By implication can't determine p_t or money supply

Economics behind Undetermined Expected Price Level

- Under interest rate rule (6), public correctly expects monetary authority will accommodate whatever quantity of money is demanded at r
- Public expects any increase in p will be equaled by increase in m
- No interest rate rule associated with a determinate price level (we have proved c).

6. An Information Advantage for the Monetary Authority

- What if monetary authority has more information than public?

  - E_{t-1} will be the expectation conditional on what the authority knows at end of t-1 and $E_{\theta,\, t-1}$ for expectation conditional on what public knows at end t-1. Where θ is a subset of what the monetary authority knows.

$$ {}_{t+i}p^*_{t-j} = E_{\theta,t-j}p_{t+I} \qquad (28) $$

Rule found optimal without information discrepancy

$$J_2 m_t = -(K_2/2K_{22})(1 - J_0 - J_1) - E_{t-1}X_t \qquad (31)$$

$$J_2(m_t - E_{\theta,t-1}m_t) = -E_{t-1}X_t + E_{\theta,t-1}X_t \qquad (32)$$

$$P_t - E_{\theta,t-1}p_t = X_t - E_{t-1}X_t \qquad (33)$$

- Given the rule of 31, distribution of K does not depend on information discrepancy. From (17) the same is true for Y.
- Now for distribution of p_t. Get equations (21) and (22)

- Loss attained under rule by 31 does not depend on information discrepancy.
- Monetary can do as well under an information discrepancy as if there was none.
- Use of rule 31 gives that the distribution of p does not depend on θ, or information discrepancy
- Loss attained under rule 31 does not depend on information discrepancy
- Monetary authority can take advantage in a limited way.
- How the distribution of Y depends on the rule for m.

- $Y_t = \alpha_1 k_{1-t} + \alpha_2(p_t - E_{\theta,t-1}p_t) + u_{1t,}$ *(35)*

- $E_{\theta,t-1}y_t = \alpha_1 E_{\theta,t-1}k_{t-1} + E_{\theta,t-1}u_{1t}$ (36)

- At end of t-1 , $E_{\theta,t-1}y_t$ is unaffected by choice of m by (36).
- To have this work , monetary authority must know exactly how the public's information differs from its own.
- Similar conclusions for distribution of k_t , however its expected variance is not minimized by rule (31).
- Structure gives rise to non-trivial dynamic problem.

Conclusions

- Under rational expectations the three assumptions have been proved.
- Rational expectations must be taken seriously
- Results concerning countercyclical macroeconomic policy are fairly robust to alterations of other features of the model such as AD, and portfolio balance condition.
- Under Autoregressive expectations there is no unique period by period equilibrium if interest rate is pegged.